Nonlinear analysis – Assignment 6

**Problem 1:**

**Problem 2:**

**Step 1:**

For this first step, the stiffness matrix is calculated using the tangent element stiffness matrix of a displacement-based fiber beam-column element. It is then compared with the stiffness matrix calculated for an elastic beam element.

To compute the stiffness matrix using the tangent element stiffness of a displacement-based fiber beam-column element, a numerical integration is used to obtain a numerical estimate of the integral: [N/m].The Gauss-Lobato numerical integration is used. Five integration points are considered. The conditions which satisfy all error partial derivatives for five integration points are:

r=[-1,-sqrt(21)/7,0,sqrt(21)/7,1]

ω=[0.1, 49/90, 32/45, 49/90, 32/45, 0.1]

B is calculated as follows:

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To calculate the integral, the domain is normalised from [0,L] to [-1,1], using a coordinate transformation: and .

The cross-section is considered constant along the length of the cantilever. Moreover, the stress in fibers is constant and equal to 1 MPa.

Therefore in N/mm:

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Description générée automatiquement

Computing the stiffness matrix for an elastic beam element, we get in N/mm:

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Description générée automatiquement

Computing the error between both stiffness matrix, we get an error of 1%.

The minimum number of fibers to have an error lower than 2% is 8 fibers (error of 1.562%).

The element resisting force Q is also using the Gauss Lobato numerical integration:

[N]

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We can verify that Q1=-a\*b=-200\*200=40000 N and Q4=a\*b=40000 N, and that the remaining entries are equal to 0 when the stress in the fibers is equal to 1 MPa.

**Step 2:**

For this second step, the nodal force and the displacements are solved using a Newton-Raphson algorithm.

The external load applied on the cantilever is applied in 30 steps. At each step, the load is applied with a maximum number of iterations of 16:

1. The tangent stiffness matrix is calculated for the free degree of freedom of the cantilever.
2. The residual force resulting from the difference between the external force applied and the element resisting force vector is calculated (only for the free degrees of freedom)
3. From this, we can solve the equation K\_ff\*q\_f=R, and find the increment of displacement of the nodes.
4. From the displacements of the nodes, the section deformation is calculated at each point of the Gauss Lobato integration.
5. From this, the strains and stresses at each point of the Gauss Lobato integration are calculated.
6. With the new stresses in the fibers, the element resisting force vector is recomputed.
7. The residual force is calculated again for the free degree of freedom of the element.
8. If the tolerance is reached, the next step of the external load applied is performed.

From this, the maximum displacement for the maximum load applied are obtained:

* Maximum displacement: 47.5 mm
* Maximum load: 450000 kNUne image contenant texte, capture d’écran, Tracé, ligne

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  Description générée automatiquement

The displacement of the far-end of a cantilever , due to a point load acting at its end is written as: 45 mm. This deformation value is close to the one obtained with the Gauss-Lobato integration method. The difference can be due to the approximation made using the Gauss-Lobato integration method.

**Step 3: Adding an elastic-perfectly plastic material**

An elastic perfectly plastic material is now considered:

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Implementing this new material, the maximum displacement for the maximum load applied are obtained:

* Maximum displacement: 77.86 mm
* Maximum load: 450000 kN

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From the graph above (load as a function of the displacement), the first fiber starts yielding when the graph is no more linear, which corresponds to Vend,y =315000 kN.

Qy=Vend,y \*L=315\*2=630 kNm

My= σy \*Wel= σy\*

There is a difference of 33%, which may be due to approximations made with the Gauss-Lobato integration method.